Category V (Grades 11&12)

1. Calculate : $\sqrt[3]{2\sqrt{5}} \cdot \sqrt[6]{\frac{16}{5}}$

   A) 4 B) 2 C) 5 D) 3

2. Find the sum of the solutions of the equation $\left(x^2-1\right)\cdot \log_{12}(1-x)=0$

   A) 4 B) 0 C) -1 D)1

3. Simplify the: $8^{\frac{2}{3}\log_2\sqrt{3+\sqrt{2}}} \cdot 27^{\frac{1}{3}\log_3(3-\sqrt{2})}$

   A) 15 B) 7 C)$\log_2 3$ D)$\log_3 2$

4. In a cone whose base radius is 4 and height 6, a cylinder with the largest volume is drawn. Find the height of the cylinder.

   A) 2 B) 2.5 C) 3 D) 4

5. Find $f'(1), if\ f(x)=\frac{\ln(x)}{e^x}$

   A) 1/e B) e-1 C) 2e D) $\sqrt{2}$

6. The President of Dollarstan is deciding between two income tax plans. According to one of the plans, all residents would pay tax equal to 10% of their yearly income (if this income is positive). According to the other plan, the first 150,000 D-dollars of a resident's yearly income would not be taxed, and the tax (if any) would equal 16% of any yearly income over 150,000 D-dollars. The President cannot decide which tax plan to propose because his own tax under either plan is the same. What is the yearly income (in D-dollars) of the President of Dollarstan? Note that this income is a positive number.

   A) 400 000 B) 240 000 C) 320 000 D) 480 000

7. Replace the asterisks with digits so that the multiplication below is correct. The product is ….

$$
\begin{array}{rcccccccc}
 & & & & * & * & * & * & * \\
 & & & \times & & * & * & * & 1 \\
\hline
 & & & & 3 & 3 & 3 & 3 & 7 \\
 & & & * & * & * & * & * & \\
 & & * & * & * & * & * & & \\
+ & * & * & * & * & * & & & \\
\hline
 & * & * & * & * & 2 & 0 & 0 & 9 & *
\end{array}
$$

A) 128720097 B) 182720097 C) 187220097 D) 172720097

8. Write the expression $\frac{a^3-8b^3}{a^2-4b^2} : \frac{a^3+4ab^2+2ba^2}{a^3+2ba^2}$ in the simplest form.

A) $2a$ B) a C) $-a$ D) $-2a$

9. Find the limit $\lim_{x\to+\infty} \frac{\sqrt{x^2+2x+5}+\sqrt{x^2-2x+5}}{2x+1}$

A) 0 B) -1 C) 1 D) 2

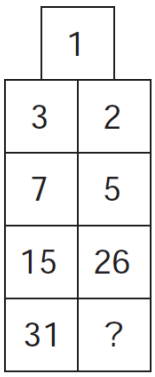
10. For any integer $n \geq 9$, the value of $\frac{(n+2)!-(n+1)!}{n!}$ is always which of the following?

A) a multiply of 4 B) a prime number
C) a perfect square D) a multiply of 10

11. If $2sinx - 5cosx = 0$, find $\frac{3cosx+2sinx}{sinx+cosx}$

A) 3/7 B) 2/5 C) 16/7 D) 0

12. Which number should replace the question mark?



A) 64 B) 123 C) 677 D) 942

13. In the equations below, the functions of ■, ⊛ and Δ are given. Accordingly, which of the following numbers does the question mark stand for in the fourth equation?

$$I. a \circledast b = 2 \cdot a \blacksquare b$$
$$II. a \blacksquare b = b \,\Delta\, 2 \cdot a$$
$$III. a \,\Delta\, b = 2 \cdot a + 2 \cdot b$$
$$IV. (2 \circledast 2) \blacksquare 2 = ?$$

(A) 72 (B) 74 (C) 76 (D) 84

14. 10 students in the class walk in the circle of mathematics, 14 - in physics, 13 - in informatics, 10 students walk in at least two circles at once, and 2 students - in all three circles. How many students do not walk in any circle if there are 27 students in a class?

A) None B) 4 C) 3 D) 2

15. If sinx= 0,6 then find sin2x=?

A) 24/25 B) 3/4 C) 10/12 D) 25/24

16. The car crosses the road between points A and B in 1 hour. The car went from A to B, meanwhile from B to A on foot. The car met the pedestrian, turned around, led him to A, and then headed back to B. It took him 2 hours and 40 minutes to get to B. At what time does the infantry go from B to A? (It is implied that the infantry moves equally and the car moves equally).

A) 4 hours B) 4.5 hours C) 5.2 hours D) 5 hours

17. Two workers have to prepare the same number of details. One worker completes $\frac{3}{8}$ part of the whole work in 6 hours, the other completes $\frac{1}{8}$ part of the work in 3 hours. They both started working at the same time. After the first one had finished his work, he helped the second one to finish the work. At what time did the workers finish the job? (It is assumed that the first worker works equally and the second worker works equally).

A) 19 hours 1 0 minutes B) 19 hours 12 minutes
C) 20 hours 6 minutes D) 18 hours 36 minutes

18. Find $\frac{sin30°}{sin10°} - \frac{cos30°}{cos10°}$

A) 0 B) 1 C) 2 D) 3

19. Solve $\lim_{x\to 0}\left(\frac{tan^2 3x}{25x^2}\right)$

A) $\frac{3}{25}$ B) $\frac{6}{25}$ C) $\frac{9}{25}$ D)6

20. Prove that any number to the power of 0(zero) is equal to 1.
Please show your work.

21. The diagonals of the axial section of the truncated cone are divided by the letter of intersection in the ratio 2:1, and the angle between them is equal to 120^0. Find the volume of the cone if the length of the diagonal is 3 meters.

A) $\frac{\sqrt{2}}{\sqrt{3}}\pi$ B) $\frac{47}{\sqrt{2}}\pi$ C) $\frac{31}{\sqrt{3}}\pi$ D) $\frac{21}{2}\pi$

22. $y = x^2$ Two points were taken on the parabola, whose abscissas are $x_1 = 1$ and $x_2 = 3$. A sharp line is drawn on these points. Write the equation of the sides that are parallel to the given angle.

A) y=4x-4 B) y=6x-9 C) y=-x D)y=0

23. Find the shortest distance from the point $\boldsymbol{A}(\mathbf{0};\ \mathbf{0})$ to a point on the graph of the function $f(x) = \sqrt{10 + x - 2x^2}$

A) 1 B) 2 C) $\sqrt{3}$ D) 3

24. It is known that the equation has no solution. Find the set of values of all such $\boldsymbol{k}$ parameters.

$$2 * 3^{2x} - (k-3) * 3^x - 2(k+1) = 0$$

A) $[-1;\ 3]$ B) $(-1;\ 1)$ C) $(-\infty;\ -1]$ D) R

25. The slope of the lateral surface of the cone is a semicircle. Find the radius of the base of the cone if the height is equal to 9.

A) $2\sqrt{3}$ B) $3\sqrt{2}$ C) $3\sqrt{3}$ D) 4